SIAM Review **Vol. 58, Issue 4 (December 2016)**

Book Reviews

BOOK REVIEWS

There are many textbooks on numerical analysis, but I have always found it difficult to find one that both satisfies me and is at the same time accessible to the students I am trying to teach. Some are too much like a cookbook, while others are too technical. Moreover, like everything else, the field continues to evolve steadily, so new entries are always welcome. This issue's featured review, by Alex Townsend, informs us about a recent effort intended for the graduate student market, *A Graduate Introduction to Numerical Methods*, by Robert Corless and Nicolas Fillion. It's a huge (though reasonably priced) book that covers lots of ground and could serve as the text for a variety of graduate numerical analysis courses. As a subtitle to the book indicates, backward error analysis plays a big role, and so does the barycentric formula for Lagrangian interpolation. I hope you enjoy Alex's very positive review of this interesting book.

In addition we have reviews of books on polynomial and semialgebraic optimization, variational analysis in Sobolev and BV spaces, plasticity, mathematical theory and numerical analysis, spline functions, and computational mathematical modeling.

Finally, there are two reviews written by me, one on an extremely technical, specialized book on domain decomposition, and the other on a book about computer science meant for the general reader.

David S. Watkins
Section Editor
siam.book.review@gmail.com

SIAM Review
Vol. 58, No. 4, pp. 795–807

Book Reviews

Edited by David S. Watkins

Featured Review: A Graduate Introduction to Numerical Methods: From the Viewpoint of Backward Error Analysis. *By Robert M. Corless and Nicolas Fillion.* Springer, New York, Heidelberg, 2013. $99.00. xxxix+869 pp., hardcover. ISBN 978-1-4614-8452-3.

I have just finished the 1.7 kilogram book *A Graduate Introduction to Numerical Methods* by Corless and Fillion. I read it chapter by chapter, covering about a chapter a week, taking it with me as I traveled to three different countries.[1] Every instructor of a graduate or advanced undergraduate numerical analysis course should consider teaching from this book. It deservedly made the ACM Computing Reviews' list of notable books and articles in 2013 [2].

What is numerical analysis? This is a difficult question, and your answer is likely to be a personal interpretation of the fuzzy interplay between "numerics" and "analysis." The book's 869-page answer is a delight. It is an inclusive celebration of Maple's symbolic manipulations and MATLAB's numerical computations, Turing's mathematical modeling [8] and Strang's linear algebra [6], Higham's backward error analysis [4] and Trefethen's practical approximation theory [7]. All the usual introductory numerical analysis topics are included such as interpolation, floating-point arithmetic, numerical linear algebra, and differential equations, as well as less common topics such as the barycentric formula, resultants, automatic differentiation, and the Lambert W function. The authors are experts on these special topics; see [1, 3]. The broad range of topics in this book shows that numerical analysis is an impressively rich and mature subject.

The book comes in four parts: Part I on error analysis, polynomials, and rootfinding, Part II on numerical linear algebra, Part III on interpolation, differentiation, and quadrature, and Part IV on differential equations. There are also helpful appendices on floating-point arithmetic, complex numbers, and introductory linear algebra. To summarize the book I made a word cloud of the text (see Figure 1), in which the size of a word indicates the frequency of its occurrence. One can also use this word cloud as a visual blurb or as a possible answer to "What is numerical analysis?"

It is a pleasure to see that the three large words are "problem," "method," and "compute": the key schema of computational mathematics! This schema is repeated hundreds of times in the book. The authors are often in investigative mode, using Maple or MATLAB or formulas to discover something interesting. A broad selection of numerical methods are described during the numerous journeys of discovery, in a manner suitable for a textbook and in a way that is very different to *Numerical Recipes* [5], which is better designed for quick reference. The unwavering andragogical style to Corless and Fillion's writing is masterful.

[1] Security control in the United States will search a backpack containing a hardback copy of Corless and Fillion's book because it is 3 inches thick. Each time one must explain the unusual carry-on item.

Publishers are invited to send books for review to Book Reviews Editor, SIAM, 3600 *Market St.,* 6*th Floor, Philadelphia, PA* 19104-2688.

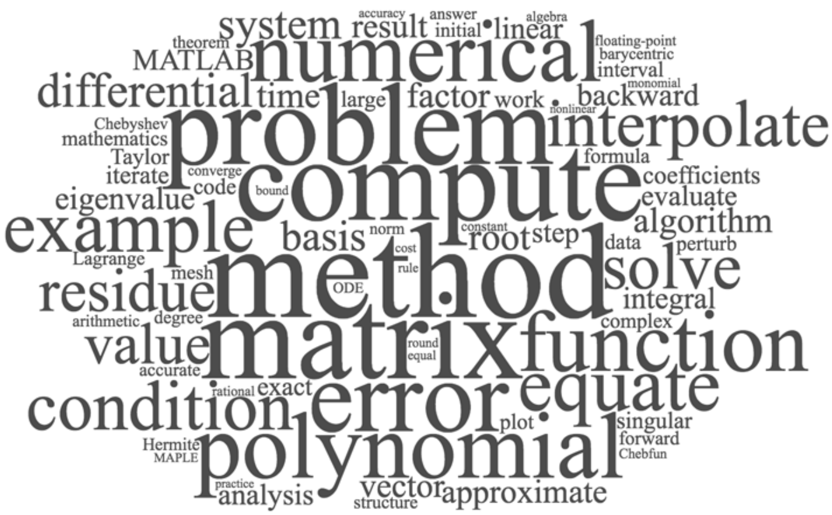

Fig. 1 *A word cloud of the text and one possible answer to "What is numerical analysis?"*

Next in the word cloud I notice the common words "polynomial" and "interpolate." This highlights one of the unique features of the book: the extensive use of polynomial interpolants and the barycentric formula, both for practical computations and for theoretical derivations. For example, how many textbooks use the barycentric formula for Hermite interpolants to derive cubic splines or to develop multistep methods? How many start by interpreting the discrete Fourier transform as a polynomial in the monomial basis evaluated at the roots-of-unity? The use of polynomial interpolants and the barycentric formula feels surprisingly fresh and modern.

Another common word is "example," which appears on average about once per page. We know that numerical analysis is useful for a diverse range of applications, and the book considers examples on the orbit of three bodies under gravity, the spread of a measles epidemic, the flight path of a spinning golf ball, and others. The example-oriented narrative is informal (see the xkcd comic on p. 543 and footnotes) and well illustrated with nearly 200 figures. This style makes the textbook engaging and friendly. An instructor could easily cover the examples given in the book during class; preferably, in a similar investigative mode.

Two other keywords that cannot be missed are "condition (number)" and "error." The book uses backward error analysis and condition numbers throughout, not only in the numerical linear algebra chapters, but also for quadrature, rootfinding, and the numerical solution of differential equations. The error analysis is professionally satisfying and will be reassuring to conscientious learners. In the preface and afterword, the authors present a thought-provoking discussion on the importance of backward error in numerical analysis, which is of independent interest and deserves attention.

There are so many intriguing features of the book that can be seen directly from the word cloud. For example, the word "residue" appears nearly 800 times because complex analysis is used extensively (another unique feature of the book), "MATLAB" is two-times larger than "Maple," and for every occurrence of "bound" there is one of "round" (suggesting a delightful balance between error analysis and floating-point arithmetic).

Aside from common words, there are also amusing maxims scattered throughout the book. Here is a selection.

It's What a Good Numerical Analyst Does Anyway. The book excels at teaching someone how to be a "good" numerical analyst. A student will get into the habit of using a computer to conjure up a theorem, a theorem to certify numerical computations, and a diagram to illustrate a concept. Each chapter also has a welcome "Notes and References" section that details places in the literature to find out more.

Which Method Is the Best Method? It's Like a Game of Rock-Paper-Scissors. In the game of rock-paper-scissors neither rock, paper, nor scissors always trumps the opponent's gesture. An analogous situation occurs for many classes of numerical techniques. The book has an unbiased didactic tone that is particularly prominent when describing quadrature rules and the numerical methods for solving differential equations. This style will keep the material relevant for several decades.

A Pupil from Whom Nothing Is Ever Demanded which (S)he Cannot Do, Never Does All (S)he Can. Each chapter has a good selection of problems that examine both the practice and the theory. One could easily set homework containing questions directly from the book, assuming the solutions are not already online. The book's charming and informative language carries over to the problems, too. For example, Problem 13.22 is on the so-called Obsessive-Compulsive Disorder Euler method. Each chapter also has a section called "Investigations and Projects" that contains more involved and open-ended questions. These will challenge the more advanced students.

I Don't Care How Quickly You Give Me the Wrong Answer. The book's central philosophy on numerical algorithms is *reliability first, cost last.* This makes error analysis and conditioning, though not necessarily algorithmic complexity, a central activity. I think this is a shame. Numerical analysts care about reliability *and* speed. As just one example, Chapter 6 on structured matrices spends the majority of time on structured backward error analysis and only briefly mentions fast matrix-vector products and matrix factorizations. Researchers usually exploit structure in matrices for computational cost, storage, and scalability to real-world problems, not for an improved theoretical error bound. I did not enjoy the *cost last* philosophy of the book.

A Problem Is "Stiff" If, in Comparison,* ode15s *Mops the Floor with* ode45 *on It. This is one of the more memorable informal definitions of a *stiff* differential equation, and highlights once again the conversational style of the book. Numerical methods for solving boundary value problems, delayed differential equations, and partial differential equations as well as Runge–Kutta and multistep methods are expertly described. There is even a detailed discussion on event handling and the role of numerical methods for chaotic systems.

One particularly illustrative example is the classic Hénon–Heiles equation, which is a nonlinear nonintegrable Hamiltonian system given by

$$\ddot{x}(t) = -\frac{\partial V}{\partial x}, \quad \ddot{y}(t) = -\frac{\partial V}{\partial y}, \quad V(x,y) = \frac{1}{2}\left(x^2 + y^2 + 2x^2y - \frac{2}{3}y^3\right). \tag{1}$$

The energy $E(t) = V(x(t), y(t)) + \frac{1}{2}(\dot{x}(t)^2 + \dot{y}(t)^2)$ is conserved during motion and serves as a way to gauge the numerical error in a method. The book gives inline MATLAB code to solve (1) for $0 < t \leq 10^5$ using `ode113`, as well as code to plot a

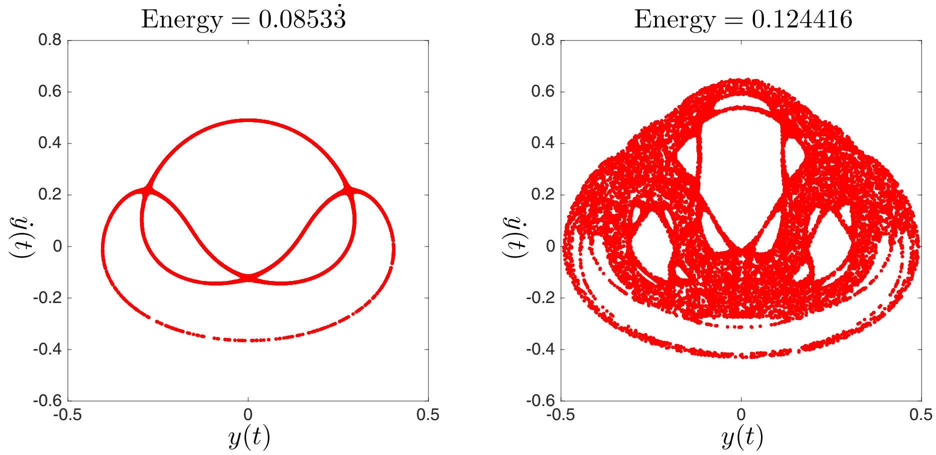

Fig. 2 *Poincaré map of the Hénon–Heiles system for energy* $32/375$ *with* $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0.2$ *(left) and energy* 0.124416 *with* $x(0) = \dot{x}(0) = y(0) = \dot{y}(0) = 0.24$ *(right; see Fig.* 12.12 *in the book). A red dot is plotted in the phase space* $(y(t), \dot{y}(t))$ *whenever* $x(t) = 0$ *and* $\dot{x}(t) > 0$*. On the right one observes chaotic behavior with "islands" of non-chaotic phase space.*

Poincaré map of the phase space using event handling. I could not resist trying it out for myself. Figure 2 shows Poincaré maps for two different energies, and on the right we see a beautiful figure showing chaotic behavior with "islands" of nonchaotic phase space. The energy was conserved up to eight digits. The inline codes in the book are an excellent starting point for students to begin further experimentation.

Corless and Fillion have clearly dedicated several years of their lives to this book. It has been worth it! The result is a carefully judged all-round numerical analysis textbook that expertly serves students and instructors. The book stands out in a crowded market. It is a textbook that numerical analysis can be very proud of.

REFERENCES

[1] D. A. Aruliah, R. M. Corless, L. Gonzalez-Vega, and A. Shakoori, *Geometric applications of the Bézout matrix in the Lagrange basis*, in Proceedings of the International Workshop on Symbolic-Numeric Computation, ACM Press, New York, 2007, pp. 55–64.

[2] *Computing Reviews*, http://computingreviews.com/recommend/bestof/notableitems_2013.cfm, 2013.

[3] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth, *On the Lambert W function*, Adv. Comput. Math., 5 (1996), pp. 329–359.

[4] N. J. Higham, *Accuracy and Stability of Numerical Algorithms*, 2nd ed., SIAM, Philadelphia, 2002.

[5] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes: The Art of Scientific Computing*, 3rd ed., Cambridge University Press, Cambridge, UK, 2007.

[6] G. Strang, *Introduction to Linear Algebra*, 4th ed., Wellesley–Cambridge Press, Wellesley, MA, 2009.

[7] L. N. Trefethen, *Approximation Theory and Approximation Practice*, SIAM, Philadelphia, 2013.

[8] A. M. Turing, *The chemical basis of morphogenesis*, Philos. Trans. Roy. Soc. B, 237 (1952), pp. 37–72.

Alex Townsend
Cornell University

An Introduction to Polynomial and Semi-Algebraic Optimization. *By Jean Bernard Lasserre.* Cambridge University Press, Cambridge, UK, 2015. $60.00. xiv+339 pp., softcover. ISBN 978-1-10763-069-7.

Consider an optimization problem of the form

$$\min f_0(x)$$

subject to

$$f_i(x) \geq 0, \quad i = 1, 2, \ldots, m.$$

This optimization problem can be extremely hard to solve, but it has various restricted forms that are more easily solvable. This book focuses on the special cases where the functions $f_i(x)$ are polynomials. This class of problems is surprisingly broad and includes many computationally challenging problems. For example, integer programming problems involving 0–1 variables can be formulated by imposing constraints $x_j^2 - x_j = 0$.

Recent developments in algebraic geometry and semidefinite programming have made the solution of small to medium sized polynomial optimization problems using the techniques described in this book computationally feasible. These techniques have already been applied in a variety of applications, and as modeling systems and semidefinite programming software improve, we can expect to see wider application of polynomial optimization.

The first part of the book is concerned with nonnegative polynomials and sums of squares. A sum of squares polynomial is a polynomial $q(x)$ that can be written in the form

$$q(x) = \sum_{i=1}^{k} q_i(x)^2.$$

Clearly, any sum of squares polynomial is nonnegative for all x. For univariate polynomials, the converse is also true. Furthermore, we can use semidefinite programming to determine whether $q(x)$ can be written as a sum of squares. A univariate polynomial $q(x)$ of degree $2d$ can be written as a sum of squares if and only if there is a symmetric and positive semidefinite matrix Q such that

$$q(x) = \begin{bmatrix} 1 & x & \ldots & x^d \end{bmatrix} Q \begin{bmatrix} 1 \\ x \\ \vdots \\ x^d \end{bmatrix}.$$

If $q(x)$ can be written in sum of squares form, then we can write

$$\begin{bmatrix} q_1(x) \\ q_2(x) \\ \vdots \\ q_k(x) \end{bmatrix} = M \begin{bmatrix} 1 \\ x \\ \vdots \\ x^d \end{bmatrix}$$

and let $Q = M^T M$. Conversely, if such a matrix Q exists, then we can find a sum of squares representation of $q(x)$ by using a Cholesky factorization $Q = R^T R$.

The semidefinite programming approach to determining whether a polynomial can be written in sum of squares form is easily extended to multivariate polynomials. Unfortunately, it is not true for multivariate polynomials that a polynomial is nonnegative if and only if it can be written as a sum of squares. However, it can be shown that in general a multivariate polynomial $g(x)$ is nonnegative if and only if it can be written as the sum of squares of rational functions. Equivalently, $g(x)$ is nonnegative if and only if there are sum of squares polynomials $f(x)$ and $h(x)$ such that $f(x) = g(x)h(x)$. Again, we can solve a semidefinite programming problem to determine whether or not $g(x)$ is nonnegative.

Using results from algebraic geometry, many semidefinite programming formulations of polynomial optimization problems and optimization problems over semi-algebraic sets are possible. The second part of this book is concerned with developing these formulations and analyzing their properties. An important result is that a hierarchy of semidefinite programming relaxation of successively larger sizes converges finitely to an exact solution of the polynomial optimization problem.

In the third part of the book, the author discusses some specialized topics and applications, including convexity in polynomial optimization, parametric polynomial optimization, convex underestimators of polynomials, and inverse polynomial optimization. Appendices cover semidefinite programming and the author's GloptiPoly software package for polynomial optimization.

Although this book has been written in the form of a textbook with exercises at the end of each chapter, the presentation is at an advanced level and depends on prerequisite knowledge of algebraic geometry and semidefinite programming. Relatively few students are likely to have the background to study this book without additional support. It is likely to be most useful as a reference for advanced graduate students and researchers working in this area. As a reference the coverage of the topic is authoritative and thorough.

The author's previous book focused on the generalized moment problem, which is in a sense dual to the problem of polynomial optimization [1]. The generalized moment problem has applications in probability theory, analysis of Markov chains, mathematical finance, and control systems. Although the current book discusses the duality between moment problems and polynomial optimization, readers interested in applications will find more material on applications in the earlier volume. Another useful feature of the earlier book is an appendix covering the required background in algebraic geometry. Many readers will find it useful to refer to both books.

REFERENCE

[1] J. B. Lasserre, *Moments, Positive Polynomials and Their Applications*, Vol. 1, World Scientific, 2009.

Brian Borchers
New Mexico Tech

Variational Analysis in Sobolev and *BV* Spaces. Applications to PDEs and Optimization. Second Edition. *By Hedy Attouch, Giuseppe Buttazzo, and Gérard Michaille*. SIAM, Philadelphia, PA, 2014. $141.00. xii+793 pp., hardcover. ISBN 978-1-611973-47-1.

This is the second edition of a book first published in 2006. It is very impressive and contains much more than the title would suggest. The first version was already a very rich textbook in analysis, going from the basics (topologies and convergences, calculus of variations, measure theory, Sobolev spaces and capacities, convex analysis) to more involved topics (BV and SBV functions, lower semicontinuity and relaxation, Young measures, Γ-convergence), illustrated by the detailed analysis of a few simple variational problems. It also contained a complete chapter on the finite element method. The second edition has been substantially enriched with more examples, new theoretical tools such as an introduction to (variational) stochastic homogenization and to optimal transportation, and with a new chapter (of more than 100 pages!) on gradient flows.

Like its previous edition, this book is divided into two parts, titled "Basic Variational Principles" and "Advanced Variational Analysis." The first part contains nine chapters, starting with the fundamentals of analysis (distributions, topologies, lower-semicontinuity, including an original approach to Ekeland's variational principle), a selection of some well-presented measure theoretic tools (Carathéodory's construction and Hausdorff measures, Young measures, capacity theory), followed by the analysis of some variational PDEs and systems in various settings (Dirichlet, Neumann, or mixed boundary conditions and transmission conditions, nonlinearities, Lamé system, critical points, obstacles) with, in fact, many more examples than in the previous edition.

This part ends with a chapter on the basics of the finite element method and an introduction to standard error estimates (note that Galerkin's approximation has already been introduced in the third section, where it is used to give a simple proof of the Lax–Milgram theorem), a detailed chapter on the spectral decomposition of the Laplace operator, and a 55-page introduction to convex duality, in Chapter 9, which could be seen as a minitextbook in itself. Indeed, this last section is very complete and nicely introduces theoretical tools such as inf-convolutions, Legendre–Fenchel duality, subdifferential calculus, and practical optimization (multipliers, KKT conditions, dual problems, linear programs). It ends with a detailed introduction to convex-concave saddle points problems and Fenchel–Rockafellar duality.

The second part of the new edition now contains eight chapters. The first one, as before, is devoted to BV and SBV functions. Many important technical points, such as the rectifiability of the level sets, are proven, at least partially. SBV functions are introduced to pave the way for the models of image segmentation and fracture growth which are introduced further on, in Chapter 14. Then follows a long and detailed chapter on lower semicontinuous relaxation, in particular, in BV or measure spaces. A short theoretical introduction is followed by a section on integral functionals with p-growth, where important concepts such as Morrey's quasiconvexity are discussed (in fact, part of this discussion is spread between this chapter and a further one on lower semicontinuity); the Young measure approach to relaxation is further discussed in great detail in a quite substantial section. The case of relaxation for problems with growth 1, in the space BV of functions with bounded variation, which requires more advanced measure-theoretical tools, is also briefly considered (and reappears two chapters later). An addition to the second edition is a very brief section on mass transportation, which introduces some essential notions and describes the Monge–Kantorovich duality. (This part might have been more appropriate as an illustration of convex duality in Chapter 9, or as a separate and more complete 18th chapter at the end of the book, where Wasserstein flows or Brenier's theorem could have been discussed.)

The next chapter introduces the notion of Γ-convergence (in metrizable spaces) and then quickly switches to useful applications such as 3D-2D limits or (variational) homogenization. Quite interesting in this new edition are the almost 30 pages on stochastic homogenization of minimization problems (with growth $p > 1$), which cover a topic rarely found in textbooks. This chapter ends with a brief description of Modica–Mortola and Ambrosio–Tortorelli approximations of perimeter/free discontinuity problems. Chapter 13 is devoted to the lower semicontinuity of integral functionals in the scalar and vectorial cases, and refines some of the results of Chapter 11 (maybe these parts could have been merged together, as there is some redundancy). It also addresses the issue of SBV functions, which are used in some of the applications studied in the next chapter. Indeed, this next part, which is very interesting, shows how the previously introduced tools are used in practical examples such as (Hencky) plasticity, fracture mechanics, and the Mumford–Shah functional. Chapter 15 is quite original. It addresses the issue of coercivity and introduces tools for the study of noncoercive variational problems. In particular, it contains a very detailed analysis of the properties of recession functions of convex functions, which are not easily found elsewhere. Next comes an introduction to some shape optimization problems. A few interesting examples are given and the most useful technical tools to deal with a few fundamental optimization problems (with respect to a domain or a potential in an elliptic PDE) are described. Finally, Chapter 17 is entirely new and is an important addition to the book. It is devoted to gradient flows, mostly in the convex case, and contains fundamental notions as well as quite original material. Four important subjects are developed. It starts with classical results (Cauchy–Lipschitz theorem, asymptotics), followed by a quite complete description of convex gradient flows, with important tools from the theory of maximal monotone operators such as Moreau–Yosida approximation, Chernoff's lemma, a version of Opial's lemma, etc. This is illustrated by PDE examples such as the Stefan problem. A following section is devoted to the asymptotics of descent trajectories of real-analytic functions, based on the Kurdyka–Łojasiewicz inequality; the recent extension to the nonsmooth case (using semialgebraic functions) is also described. A third part studies limits of sequences of gradient flow problems, in the convex case (hence, based on Mosco-convergence of functionals), with an interesting application to stochastic homogenization in diffusion equations. Eventually, gradient flows in metric spaces are rapidly introduced together with the minimizing movement approach of De Giorgi, in a very short section which refers primarily to the well-known monograph of Ambrosio, Gigli, and Savaré.

All in all, this long textbook is very complete and pleasant to read, with a progressive level of difficulty and complexity and many nice examples which illustrate the theoretical results. It contains deep and precise information on many important tools in variational analysis (functional analysis, convex analysis) and many advanced methods, together with a general overview of most of the modern techniques. It should be useful for both students and researchers, whether they need to learn or review some advanced techniques in analysis or are looking for an introduction to more recent theories.

Antonin Chambolle
CNRS
École Polytechnique

Plasticity: Mathematical Theory and Numerical Analysis. Second Edition. *By Weimin Han and B. Daya Reddy.* Springer, New York, 2013. $149. xvi+424 pp., hardcover. ISBN 978-1-4614-5939-2.

The discipline of plasticity is concerned with the study of irreversible deformations in solids. The governing equations for plastic evolution can take widely different forms depending on the phenomenological model and loading conditions considered. This renders plasticity a challenging pursuit for both the solid mechanician and the mathematician. Moreover, due to the rapid progress made in plasticity research, the mechanician has been left rather unfamiliar with the developments in the mathematical theory and likewise the mathematician with the physical background of the recent plasticity models. The book under review is an ambitious effort to fill this gap by making recent mathematical research in plasticity accessible to the nonmathematician without losing sight of both the rigor as well as the physical basis for plasticity theories. The book is unique in its broad consideration of analytical and numerical aspects of plasticity. In this second edition, important material relevant to strain gradient and single crystal plasticity theories has been added.

The book is divided into three parts. The first part, consisting of four chapters, provides a quick summary of the relevant notions from continuum mechanics and introduces several formulations of rate-independent elastoplasticity. The latter is presented first in a classical framework and then in a convex-analytic setting. After fixing the notation in Chapter 1, concepts from continuum mechanics, including kinematics, balance laws, dissipation, and linearized elasticity, are collected in Chapter 2. Next, in Chapter 3, this is followed by a survey of rate-independent elastoplastic theories within a classical framework, with an emphasis on isotropic strain gradient models of plasticity (particularly the Gurtin–Anand model) and small deformation single crystal plasticity. Chapters 2 and 3 provide only a minimalistic, although well-written, exposure of the physical aspects of plasticity to a nonspecialist, who will have to look elsewhere for more comprehensive treatments. The elastoplastic problems are restated again in Chapter 4, now within a convex-analytic framework. In this form, the problems can be formulated in more generality, such as allowing for nonsmooth yield loci, in addition to being amenable for further mathematical analysis.

The second part of the book seeks to resolve the well-posedness of the initial-boundary value problems of elastoplasticity introduced in Chapters 3 and 4. To this end, the problems are posited in terms of variational inequalities which are subsequently used to demonstrate existence and uniqueness of the solutions. The prerequisite notions from functional analysis and variational inequalities are collected in Chapters 5 and 6, respectively. An interested reader, looking for details, would have to look at several excellent texts available on these subjects. Chapters 7 and 8, combined with their numerical counterparts in Chapters 12 and 13, form the core of the book. They contain a detailed mathematical analysis of the primal and the dual variational problems of elastoplasticity, respectively. Whereas the former has displacement, plastic strain, and hardening parameters as unknowns, the latter solves for generalized stress as the unknown variable. In both the cases, the emphasis is on proving existence and uniqueness of solution to the weak formulation of classical and strain-gradient plasticity problems for polycrystalline and single-crystalline materials. Drawing heavily from their research papers, the authors have successfully managed to present a developed picture of the mathematical theory. This should be extremely valuable for both the mechanician and the mathematician.

The final part of the book is concerned with the numerical analysis of computational algorithms for solving elastoplasticity problems. The temporal and spatial variations are approximated using the finite difference and finite element methods, respectively. After succinctly recalling pertinent aspects of finite element analysis in Chapter 9, approximation of variational equations and inequalities using the finite element method is discussed in Chapter 10. The majority of the discussion is on obtaining reasonable error estimates for the finite element solution of the variational problems. Chapter 11 takes another step toward that goal by introducing semidiscrete and fully discrete approximations and establishing the relevant error estimates. Convergence under minimal regularity assumptions is also established. Finally, in Chapters 12 and 13, we come back to elastoplasticity. Chapter 12 focuses on the implementation of numerical schemes for the primal elastoplastic problem. The error estimates and solution algorithms are derived for various plasticity models and convergence of the algorithms is rigorously established. In Chapter 13, numerical analysis of the dual variational problem is undertaken. Again, several numerical schemes are introduced and analyzed. The focus of this chapter is on implementing time-discrete schemes for classical problems in plasticity.

The book is well written and carefully presented overall. It presents a wealth of useful material otherwise absent from other plasticity books. It exposes both the interested mechanician and the interested mathematician to mathematical problems in plasticity in a unified manner, albeit to be pursued in their own way. My only disappointment with the book is the absence of bibliographic notes at the end of each chapter (aside from one in Chapter 3). Their inclusion would have not only provided further reading directions and open problems, but also given an overall perspective on an active research discipline within which the contents of the book are placed.

ANURAG GUPTA
Indian Institute of Technology Kanpur

Spline Functions: Computational Methods. *By Larry L. Schumaker.* SIAM, Philadelphia, PA, 2015. $83.00. xii+412 pp., hardcover. ISBN 978-1-611973-89-1.

The book is a long-awaited sequel to two previous books on splines, [2] and [1], by the same author. The new book is a product of over fifty years of research, teaching, and collaboration with numerous scientists within and outside the field of splines. The complete bibliography comprised of over 100 pages was too long for the hard copy and has been put online; see [3]. A very valuable part of the book is a MATLAB package, *SplinePak*, which is freely available both on [3] and on Larry L. Schumaker's website [4]. For the impatient reader, I would recommend downloading the package and going straight to the examples in the book, which, admittedly, is exactly what I did. Within minutes, I had a beautiful picture of a minimal energy quadratic smooth spherical spline interpolating $f(x, y, z) = x^4 + 1.1y^4 + 1.3z^4$ at 42 vertices of a triangulation of the unit sphere; see Figure 3. Each numerical example in the text describes a problem and has a reference to the code that provides a solution. The only drawback is that the MATLAB functions in *SplinePak* are currently p-files, and thus cannot be modified. The script files are the usual m-files. A complete list of the scripts and the functions in the package is included at the end of the book.

For the more patient reader, I recommend downloading the package and getting hold of both of Larry Schumaker's previous books on splines, *Spline Functions: Basic Theory* [2] and *Spline Functions on Triangulations* [1] (coauthored with M. J. Lai). This takes us to the next truly valuable feature of the book: it is equally useful to the reader looking for algorithms to solve

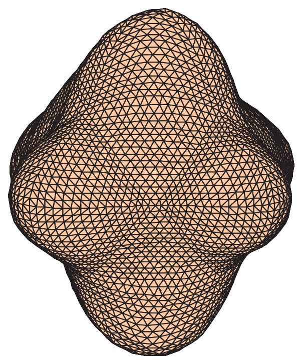

Fig. 3 *A minimal energy spherical spline.*

practical problems and to the reader interested in rigorous foundations for such algorithms. While the book itself includes few proofs, it contains rigorous statements of all theorems used, and full references to their proofs elsewhere. Most of the referenced proofs can be found in [2] and [1]. I will demonstrate this approach by summarizing the content of one section—section 5.2—titled "The C^1 Powell–Sabin Interpolant." It begins by introducing the so-called Powell–Sabin refinement of a triangulation. The following theorem states interpolating conditions sufficient to define a unique interpolating C^1 quadratic spline on the Powell–Sabin refinement. Immediately after the theorem we find a reference to section 6.3 of [1], where the proof can be found. Next, we discover all the necessary formulae to define the coefficients of the interpolating spline, followed by a reference to a code in *SplinePak* that returns a vector of the coefficients of the spline along with a figure of the spline surface. Two numerical examples that are discussed next include the maximum and root mean square errors and the convergence rate analysis. The two examples treat an interpolation problem of Franke's function on a triangulation with 36 vertices, and on several refinements of type-I triangulation. The section concludes with the statement of a rigorous error bound in Sobolev norm followed by the general idea of the proof and a reference in [1], where the proof can be found.

The material covered by the book is broad. Piecewise polynomials (splines) in one or two variables can be used to solve approximation, interpolation, data fitting, numerical quadrature, and ODE and PDE problems, and this is exactly what the book shows the reader how to do. Chapter 1 deals with univariate splines, their evaluation, and their use in interpolation, approximation, and solving two-point boundary-value problems. Chapter 2 treats similar aspects of bivariate tensor-product splines. Theoretical foundations are covered in [2]. In Chapter 3 we learn how to deal with triangulations computationally. Chapter 4 is essential for understanding the rest of the book. It covers foundations of the Bernstein–Bézier representation of bivariate polynomials and splines. The first application of bivariate splines—Hermite interpolation—is demonstrated in Chapter 5, followed by scattered data fitting in Chapters 6, 7, and 8. Elliptic PDEs of orders two and four are the focus of Chapter 9, where the Ritz–Galerkin method is implemented with bivariate macroelements. Chapters 9 and 10 cover spherical splines and their applications. Theoretical foundations for Chapters 3–5, 10, and 11 can be found in [1]. The bibliography included at the end of the book contains referenced books only, while the online one contains research papers and other resources.

Yet another aspect of this book that makes it very attractive is the fact that the research content is fully up to date. For example, the classical subject of computing triangulations includes recent developments on triangulations with hanging vertices. Solving a classical PDE problem using the finite element method bypasses the reference triangle by means of the Bernstein–Bézier representation and uses finite elements of higher degree and smoothness. Each chapter ends with remarks and historical notes, where all original sources are cited and additional research articles are referenced.

The book is an invaluable resource for many different types of readers, including researchers in the field of numerical analysis, applied mathematicians, computer scientists and engineers, and graduate students, as well as computational specialists from other sciences who are willing to apply splines to their fields.

REFERENCES

[1] M.-J. Lai and L. L. Schumaker, *Spline Functions on Triangulations*, Cambridge University Press, Cambridge, UK, 2007.

[2] L. L. Schumaker, *Spline Functions: Basic Theory*, Wiley Interscience, New York, 1981.

[3] www.siam.org/books/ot142.

[4] www.math.vanderbilt.edu/~schumake/.

Tatyana Sorokina
Towson University

Computational Mathematical Modeling: An Integrated Approach Across Scales. *By Daniela Calvetti and Erkki Somersalo.* SIAM, Philadelphia, PA, 2013. $71.50. x+224 pp., softcover. ISBN 978-1-611972-47-4.

The book *Computational Mathematical Modeling: An Integrated Approach Across Scales* is divided into nine chapters, where the first four are devoted to deterministic models which actually boil down to solving ordinary differential equations. The remaining five chapters deal with mathematical models that include noise, i.e., random numbers as a stochastic element.

The material is presented in a writing style typical of mathematical textbooks, with many theorems and formal definitions that will probably be easy to grasp for math majors. Each topic is presented so as to convey the basic principles without going into too much detail. For example, Chapter 5 on random variables and distributions could easily fill an entire book. Hence, in general the surface of each topic is merely scratched and, after an introduction of the governing equations, examples are provided for possible solution strategies. Some readers will find the MATLAB exercises, which are scattered throughout the book, useful even though their solutions are only provided as code snippets, so the student will have to fill in the gaps. No introduction to MATLAB is provided. This comes as somewhat of a disappointment as the word “computational” in the book title suggests more than just a bunch of MATLAB snippets. In terms of modeling, the authors stay almost completely in the realm of pure mathematics and only rarely are examples given from other areas. For example, in Chapter 7 the stochastic simulation of chemical reactions is discussed though this, however—to the reviewer’s knowledge—is not really used for calculating reaction kinetics in chemistry. Chemists use the Arrhenius equation for that. Markov processes and the standard predator-prey model used in biology are covered to some extent in Chapter 8.

Each chapter ends with a number of exercises which are almost all purely mathematical. Some of them involve writing MATLAB code. Considering that neither hints nor solutions are provided to the exercises, I have doubts that they will be of much use to the average student, because most of them are hard to carry out after having read only the preceding chapter. There is a useful subject index and a good bibliography at the end of the book. Each chapter additionally has its own bibliography, which is quite useful. However, the chapter bibliographies are not very extensive: most references cite other books and there are hardly any references to primary source papers.

All in all, this reviewer thinks that this book is a good read, is technically sound, and can be recommended for beginning to advanced graduate students who want to become acquainted with several basic ideas in mathematical modeling. The book will probably be most useful to math majors due to its presentation style. I doubt that the book will be of much use for students majoring in subjects other than math, despite what the back cover suggests, because very few applications from other fields are discussed. Students fluent in the MATLAB scripting language will find the corresponding exercises and examples helpful.

Martin O. Steinhauser
University of Basel

Dirichlet–Dirichlet Domain Decomposition Methods for Elliptic Problems: *h* and *hp* Finite Element Discretizations. *By Vadim Glebovich Korneev and Ulrich Langer.* World Scientific, Hackensack, NJ, 2015. $128.00. xx+463 pp., hardcover. ISBN 978-981-4578-45-5.

The finite element method for solving elliptic boundary value problems first gained popularity some fifty years ago. The use of iterative methods to solve the resulting systems became imperative as their size grew over time. In order to solve the huge systems in parallel, domain decomposition (DD) techniques were created. These are iterative methods that proceed by solving smaller problems on subdomains. Although they can stand on their own as iterative methods, they are more effective when used as preconditioners for the conjugate gradient method. For maximum efficiency, the subdomain problems are also solved iteratively, typically by a preconditioned conjugate gradient method. A major objective is to achieve optimal computational complexity by producing a method for which the number of iterations is bounded as the element size $h \to 0$. (In the hp version, the polynomial degree $p \to \infty$ as well.) For this it suffices to keep the condition number of the preconditioned system bounded as $h \to 0$ (and, in the hp version, $p \to \infty$). The right choice of preconditioners for the subdomain problems is crucial.

The DD literature is huge, having exploded in the past thirty years. DD techniques come in a variety of flavors; for one thing, the subdomains can be overlapping or not. The overlapping methods are easier to understand and implement, but the nonoverlapping methods may be more useful. For example, if the coefficients of the differential operator have jump discontinuities, it is better to have subdomain boundaries at the jumps and not have any overlap.

Within the nonoverlapping methods there are many variants. The book under review focuses on a specific class of DD methods called Dirichlet–Dirichlet after the type of boundary conditions that are used in the subdomains. This is a book for experts only. The entire jargon of finite element and iterative solver theory is used with little or no explanation. The authors expect that the reader knows what these things mean.

The book has one chapter on overlapping methods, but the main focus is on nonoverlapping Dirichlet–Dirichlet DD techniques. There are separate chapters on the 2D and 3D cases, as there are significant differences in the way they are organized. A major objective of the authors was to present the hp theory, which is more recent and more complicated, along with the older and better-established h theory. Thus, there is one chapter each on h version in 2D, h version in 3D, hp version in 2D, and hp version in 3D. By now I have mentioned most of the book's chapters, and this is the main structure of the book, but there are a few other supporting chapters as well.

This book will certainly be useful to individuals pursuing research on DD methods. It could have benefited from some aggressive copyediting by a person whose native language is English.

David S. Watkins
Washington State University

The Computing Universe: A Journey through a Revolution. *By Tony Hey and Gyuri Pápay.* Cambridge University Press, Cambridge, UK, 2015. $85.00. xvi+397 pp., hardcover. ISBN 978-052-1766-45-6.

This is a popular book on computer science meant to be intelligible to high school and university students, as well as general readers. The goal is to get young people excited about the field. Technicalities are kept to a minimum, and almost all of the chapters could be read by just about anybody.

The book covers a lot of ground, from the beginning of the electronic computing era right up to the present. Here is a sample of chapter titles: "The Software is in the Holes," "Mr. Turing's Amazing Machines," "Computing gets Personal," "Licklider's Intergalactic Computer Network," "The Dark Side of the Web," "The End of Moore's Law," "The Third Age of Computing." There are 17 chapters in all. They do not need to be read in order: The reader can turn to a random chapter, or even a random page, and find something interesting. On almost every page there is at least one box or sidebar with information about a famous person in the field, or an interesting anecdote, or a cartoon. On pages 82–83 you can read about three space launch catastrophes that were caused by software errors. On page 117 you will learn about Kurt Gödel's interview for U.S. citizenship, accompanied by character witnesses Einstein and Morgenstern. Fortunately, the interview was successful even though Gödel had noticed inconsistencies in the constitution. On page 359 is a photograph of Turing's work area at Bletchley Park with his tea mug chained to the radiator so that it won't be stolen. I guess these old items stand out to this old reader, but there is lots of newer material as well.

I've found this book to be interesting and informative, and I'm still reading. When I'm done, I'll leave it lying around somewhere where the 16-year-old in my household might pick it up and start reading it. This assumes she can tear hers eyes away from her smart phone.

David S. Watkins
Washington State University