Lecture 3

The Limits of Predictability

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Plan

Primer: High Resolution Climate Projection

Part I – The Perils of Model Error


Part II – The Limits of Post-Processing


Outlook: What next?
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Primer

High-Resolution Climate Projections
Climate Change Is Real

Sources: genlovers.blogspot.com; coastalcare.org; weather.com; uttendorf.com
Climate Change Is Man-Made

Sources: driverside.com; china-acm.com; interestingenergyfacts.blogspot.com; climateaudit.org
MIND THE GAP
Source: tropical-rainforest-animals.com/global-warming-pictures.html
The Basic Question:

Starting point: Climate change is real!

But what does climate change mean at a local level?

Global: global mean temperature, average sea level rise, maybe melting of arctic ice.

Local: pretty much what happens at your doorstep.
One would like to know how the local climate changes because policy is made at the local level.

Make provisions:

• Adaption: flood walls, water provision, etc.

• Mitigation: implement changes and ideally stop bad things from happening.
Concrete example: UKCP09

The *United Kingdom Climate Impacts Program*’s UKCP09 project aims to answer questions about the local impact of global climate change by making high resolution forecasts of the local climate out to 2100.

The declared aim and purpose of UKCP09 is to provide decision-relevant forecasts, on which industry and policy makers can base their future plans.
The launch document says:

‘The projections have been designed as input to the difficult choices that planners and other decision-makers will need to make, in sectors such as transport, healthcare, water-resources and coastal defences, to ensure that UK is adapting well to the changes in climate that have already begun and are likely to grow in future.’ (Jenkins et al 2009, 9)
The launch document says:

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In concrete terms:
Probabilistic predictions are given on a 25km grid for finely defined events such as
- changes in the temperature of the warmest day of a summer
- precipitation of the wettest day of the winter
It is predicted, for instance, that under a medium emission scenario the probability for a 20-30% reduction in summer mean precipitation in central London in 2080 is 0.5
10% probability level
Very unlikely to be less than

50% probability level
Central estimate

90% probability level
Very unlikely to be greater than

Summer

Source: UKCP09 Briefing Report, p. 32.
How are these Result Generated?

1. GCM: approx. 300 runs of HadCM3 or HadSM3.

2. Post-process these results to obtain local predictions on 25km grid.
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2. Post-process these results to obtain local predictions on 25km grid.

Question: Are these results trustworthy?
Two points matter:

1. Strong simplifications are made to construct the model. So we are faced with **model error**.
2. As a matter of fact the dynamics is **nonlinear**.
Central Question:

(a) Are the outcomes of *nonlinear models with structural model error* trustworthy and reliable?

(b) Can the outputs of *nonlinear models with structural model error* form the basis of responsible policy making?
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(a) Are the outcomes of nonlinear models with structural model error trustworthy and reliable?

(b) Can the outputs of nonlinear models with structural model error form the basis of responsible policy making?

Preview: no and no.
Part I

The Perils of Model Error
The Question

At the most general level:

Models often aren’t exact replicas of their target systems.

How bad is it if our model is not an exact replica of the target system?
The Question

At the most general level:

Models often aren’t exact replicas of their target systems.

How bad is it if our model is not an exact replica of the target system?

→ It’s pretty bad.
The Question

A bit more specifically:
A dynamical model has **structural model error** (SME) if its time evolution is relevantly different from that of the target system, possibly due to simplifications and idealisations.

**Question:** what are the consequences of SME for a model’s predictive capacity?
Take-Home Message - Part 1

If chaotic models have even the slightest SME, their capacity to make meaningful forecasts is seriously compromised.

This has dramatic consequences for our ability to make the kind of forecasts about the future that policy makers would like to have.
Attention: *not* the same old story.

So far chaos has been studied in connection with uncertainty about *initial conditions*.

We ask what happens if we are uncertain about the correct *model structure*.

These are completely different problems!
Butterfly effect:
Error in initial conditions
Butterfly effect:
Error in initial conditions

Hawkmoth Effect:
Error in the model structure (equations)
(Erica Thompson)
Take-Home Message – Part 2
We can mitigate against the butterfly effect by making probabilistic predictions rather than point forecasts.
This route is foreclosed in the case of the hawkmoth effect: nothing can mitigate against that effect!
So structural model error and not uncertainty in the initial conditions is what truly limits predictive power.
Or: butterflies are pretty; hawkmoths are ugly.
Let’s get started
A Primer on Models

Dynamical system \((X, \phi_t, \mu)\)
A Primer on Models

Dynamical system $(X, \phi_t, \mu)$
A Primer on Models

Dynamical system \((X, \phi_t, \mu)\)
Simple example: stone falling from tower
Simple example: stone falling from tower

\( (X, \phi_t, \mu) \)
Simple example: stone falling from tower

\[ (X, \phi_t, \mu) \]
Simple example: stone falling from tower

\[(X, \phi_t, \mu)\]

Lebesgue Measure
Difficult example: global climate model
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Difficult example: global climate model

Literally 10,000s of climate variables for the entire world

\[(X, \phi_t, \mu)\]
Difficult example: global climate model

Literally 10,000s of climate variables for the entire world

\((X, \phi_t, \mu)\)

The evolution of these variables over time
Difficult example: global climate model

Literal 10,000s of climate variables for the entire world

\((X, \phi_t, \mu)\)

The evolution of these variables over time

The so-called invariant measure of the dynamics
Locating the Issues

Dynamical system \((X, \phi_t, \mu)\)
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Initial Condition Error (ICE)
Locating the Issues

Dynamical system \((X, \phi_t, \mu)\)

Initial Condition Error (ICE)
Locating the Issues

Initial condition error

Butterfly Effect
Locating the Issues

**SME:** the time evolution of the model differ from the time evolution of the system under study:

\[ \phi^S_t = \phi^M_t + \delta_t \]

True dynamics  Model  “Difference”
Locating the Issues

Dynamical system \((X, \phi_t, \mu)\)
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Locating the Issues

Structural Model Error

Hawkmoth Effect
ICE versus SME
Meet Laplace’s Demon

1. Unlimited computational power
2. Unlimited dynamical knowledge
3. Unlimited observational power

(Laplace 1814)
The Demon knows everything. In Laplace’s own words: ‘nothing would be uncertain and the future, as the past, would be present to [his] eyes’.

So the Demon’s model of the world’s climate would be trustworthy because it provides the full truth.

*But what happens if we are less capable than the Demon?*
Meet the Senior Apprentice

1. Unlimited computational power
2. Unlimited dynamical knowledge
3. No unlimited observational power
Meet the Senior Apprentice

1. Unlimited computational power
2. Unlimited dynamical knowledge
3. No unlimited observational power

In other words, the Senior Apprentice only has noisy observations.
How could the limitation of not having unlimited observational power be overcome?

Reply:

Initial Condition Ensemble
That is, she puts a probability distribution over an approximate initial condition.
Prediction? Time Evolution?
Generate probabilistic predictions by moving the initial probability distribution forward in time:
Implications for prediction? They figure that in non-linear systems we expect the probability distribution to disperse.
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Why dispersion?
Why dispersion?
Distributions become *uninformative* as time passes, but they *do not become misleading*.

The Senior Apprentice realises that this is the limitation that she has to accept.

It is the price to pay for not having unlimited observational power.
Or: butterflies are pretty; hawkmoths are ugly.
Meet the Freshman Apprentice

1. Unlimited computational power
2. No unlimited dynamical knowledge
3. No unlimited observational power
The Freshman Apprentice now claims he can do everything that the Senior Apprentice can do, his additional limitation notwithstanding.
Recall: The Freshman can’t formulate the exact dynamics of a system.

Reaction: Distortions and idealisations of all kind are acceptable as long as the resulting model is close enough to the truth.

This is the closeness-to-goodness link.

→ This is a crucial part!
That is, the Freshman claims that his probabilistic predications are as good as the Senior Apprentice’s because he can rely on the closeness to goodness link.

Notice: Real-world scientists bear striking similarities to the Apprentice.

Question: is the Apprentice right?
No way!
Population density:

\[ \rho = \frac{\# \text{ fish} / m^3}{\# \text{ max fish} / m^3} \]
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Hence: \( \rho \in [0,1] \)
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Hence:  \( \rho \in [0,1] \)

Model:

\[ \rho_{t+1} = 4\rho_t (1 - \rho_t) \]
\[ \tilde{\rho}_n = 4\tilde{\rho}_r (1 - \tilde{\rho}_r) \left[ (1 - \varepsilon) + \frac{4}{5} \varepsilon \left( \tilde{\rho}_r^2 - \tilde{\rho}_r + 1 \right) \right] \]
\[ \tilde{\rho}_{t+1} = 4 \tilde{\rho}_t (1 - \varepsilon)(1 - \tilde{\rho}_t) + \varepsilon \frac{16}{5} \tilde{\rho}_t (1 - 2 \tilde{\rho}_t^2 + \tilde{\rho}_t^3) \]

where \( \varepsilon = 0.1 \)
The Apprentice remains defiant:

Green – Apprentice and Red - Demon
Mathematically:

\[ \rho_{t+1} = 4 \rho_t (1 - \rho_t) + \text{small perturbation} \]

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One step error: 0.001
Mathematically:

\[ \rho_{t+1} = 4\rho_t (1 - \rho_t) + \text{small perturbation} \]

\[ \tilde{\rho}_{t+1} = 4\tilde{\rho}_t (1 - \varepsilon)(1 - \tilde{\rho}_t) + \varepsilon \frac{16}{5} \tilde{\rho}_t (1 - 2\tilde{\rho}_t^2 + \tilde{\rho}_t^3) \]

One step error: 0.001

Closeness-to-goodness link: this is close enough and predictions are reliable.
\[ \tilde{p}_{\alpha} = 4\tilde{p}_{i}(1-\tilde{p}_{i}) \left[ (1-\epsilon) + \frac{4}{5} \epsilon \left( \tilde{p}_{i}^2 - \tilde{p}_{i} + 1 \right) \right] \]
They all do the Calculation ....
If you use your model to offer predictions you get it completely wrong!

- You regard things that never happen as very likely.
- You regard things that happen very often as unlikely.
And if you use your model to offer bets (or insurance policies) on certain events, you are losing money!

Probability $p$ on event $E$: $p(E)$

Odds on $E$: $o(E) = 1/p \rightarrow$ pay-out if $E$ occurs
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Probability $p$ on event $E$: $p(E)$
Odds on $E$: $o(E) = 1/p \quad \rightarrow \text{pay-out if } E \text{ occurs}$

Example: coin $p$ or heads is $\frac{1}{2}$.
Odds on heads is 2.
If you bet £1 on heads and head occurs you get £2 back.
“Lower Half” against “Upper Half"
Model: \( p(U) = 0 \) and \( o(U) \to \infty \)

System: \( p(U) = 1 \)

So U happens with probability 1 and you have to pay out infinite gains!
Question: is this a special case?
Relative Entropy of 2048 initial distributions (t=8)
The Pond Casino
Nine punters with £1000 each. In every round they bet 10% of their wealth on events with probability in the interval:

1\textsuperscript{st} Punter: [1/2, 1]
2\textsuperscript{nd} Punter: [1/4), 1/2)

... 
9\textsuperscript{th} Punter: [0, 1/256)

How are they doing?
Punters' wealth

Time (Number of rounds played)
Result:

- 7 out of the 9 punters make enormous gains!
- The casino runs up huge losses.

→ Insurance companies …

But: is this just a bad “bad luck event”?
Again

Question: is this a special case?
Time to bust for 2048 casinos:
Conclusion:
Even though the model is very close to the truth, it provides ruinous predictions!
Hence: If chaotic models have even the slightest model error, their capacity to make meaningful (and policy relevant!) probabilistic forecasts is lost.
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Even though the model is very close to the truth, it provides ruinous predictions!
Hence: If chaotic models have even the slightest model error, their capacity to make meaningful (and policy relevant!) probabilistic forecasts is lost.

The closeness-to-goodness link is wrong!
The failure of the closeness-to-goodness link gives raise to the **hawkmoth effect**: the smallest deviation in model structure leads to completely different results, both for deterministic *and* probabilistic forecasts.
Therefore: an Initial Condition Ensemble and the closeness to goodness link are not an adequate means to deal with structural model error.
Or: butterflies are pretty; hawkmoths are ugly.
Reinventing the wheel?
Feigenbaum’s classical discussion:

$$\rho_{t+1} = \alpha \rho_t (1 - \rho_t)$$

Parameter:
$$\alpha \in [0, 4]$$
Time series for different parameter values:

\[ x_{n+1} = 2.95 x_n (1 - x_n) \]

\[ \alpha = 2.95 \]
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\[ x_{n+1} = 2.95x_n(1-x_n) \]

\[ \alpha = 2.95 \]

\[ x_{n+1} = 3.5x_n(1-x_n) \]

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\[ x_{n+1} = 3.5x_n(1-x_n) \]

\[ \alpha = 3.5 \]

\[ x_{n+1} = 4x_n(1-x_n) \]

\[ \alpha = 4 \]
This is a study of parameter variation.

It provides information about what happens if we are uncertain about parameter values.

But: it provides no information about what happens when we are uncertain about the model structure.

What if the true equation is not exactly

\[ \rho_{t+1} = \alpha \rho_t (1 - \rho_t) \]?
Overselling an example?
Recall our conclusion: the closeness to goodness link is not an adequate means to deal with structural model error.

Why is this a general problem and not just a problem of our example?

There is an elaborate mathematical theory of structural stability:
Andronov and Pontrjagin, Peixoto, Palis, Smale, Mañé, Hayashi.
But:

Stability proofs are forthcoming only for two-dimensional flows!

But that is a very special kind of system!

In general the situation is more involved:
Axiom A: the system is uniformly hyperbolic.  
Strong transversality condition: stable and unstable manifolds must intersect transversely at every point.  
Palis and Smale (1970) conjectured that a system is structurally stable iff it satisfies Axiom A and the strong tranversality condition.  
Proofs:  
Mañé (1988) for maps  
Hayashi (1997) for flows.
What do Axiom A and the strong transversality condition mean for physical models?
What do Axiom A and the strong transversality condition mean for physical models?

Physical models? What are you talking about?
But:
Smale (1966): structural stability is not generic in the class of diffeomorphisms on a manifold: the set of structurally stable systems is open but not dense.
Smith (2002) and Judd and Smith (2004): if the model’s and the system’s dynamics are not identical, then ‘no state of the model has a trajectory consistent with observations of the system’ (2004, 228).
Minimal conclusion: shift of the onus of proof!

Those using non-linear models for predictive purposes owe us an argument that they are structurally stable, not *vice versa*!
Part II

The Limits of Post-Processing
Fact: HadCM3 involves strong idealising assumptions
→ It has structural model error.
→ Discussion in Lecture 2.

UKCP09 acknowledges the presence of model error and suggests a way of dealing with it.
The message is that the uncertainties due to SME can be estimated and taken into account in projections.

They do so with a complex and involved computational scheme.

→ “Long paper”
→ Here focus only on the crucial assumptions.
First introduce a so-called discrepancy term:

\[ c = \varphi_{t^*}^S(x_0; \alpha^*) + d \]
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\[ c = \varphi_{t^*}^{S}(x_0; \alpha^*) + d \]

Where:
- \( c = \) true climate (in the world)
- \( \varphi = \) model dynamics
- \( x_0 = \) initial condition
- \( \alpha^* = \) parameters that best simulate the target
- \( d = \) discrepancy term
The discrepancy term tells us

‘what the model output would be if all the inadequacies in the climate model were removed, without prior knowledge of the observed outcome’ (Sexton et al 2012, 2515).
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‘what the model output would be if all the inadequacies in the climate model were removed, without prior knowledge of the observed outcome’ (Sexton et al 2012, 2515).

And then follows the crucial assumption:

‘Our key assumption is that sampling the effects of structural differences between the model chosen for the PPE and alternative models provides a reasonable proxy for the effects of structural errors in the chosen model relative to the real world.’ (ibid., 2516)
That is: UKCP09 compares the output of the model to 12 other models. The claim then is that measuring the average distance of HadSM3 to a set of different models yields a similar result as measuring its distance to the real world – hence, $d$ can be determined by measuring by how much HadSM3 diverges from those other models.

Furthermore: the error in $d$ is assumed to be Gaussian.
How good are these assumptions?

1. Is comparison with a model ensemble a good proxy for comparison with the world?

2. Is the error Gaussian?
Basic line of argument:

‘Indeed, the multimodel ensemble mean has been shown to be a more skilful representation of the present-day climate than any individual member’ (ibid.)

‘the structural errors in different models can be taken to be independent’ (ibid.)

But:

- Is ‘more skilful’ close to being ‘skilful’?
- Independence?
Recall the model ensemble on global mean temperature.

Models show warming but average temperatures vary tremendously. The magnitude of the error in the global mean in a hindcast casts significant doubt on the viability of the informativeness assumption on a 25 km forecast to the end of this century.
Further Assumptions in the scheme:
- Use of an Emulator
- Choice of a trapezoid prior probability distributions:
  - Principle of indifference
  - Robustness of posteriors
- Downscaling
- Initial condition uncertainty
Outlook

What next?
Think Different

For Science:
• Don’t aim to reduce uncertainty.
• Instead better understand, classify and communicate uncertainty.

For Policy:
• Renounce the first-predict-then-act rule.
• Decisions can be made under uncertainty.
Shifting Paradigm
Expert Elicitation

Concept diagram of climate modeling

Sources: theresilientearth.com; eofdreams.com
A Way Forward

- Better actionable information at local level.
- Cheaper and faster.
- Robust decision-making.
- Informs resilience planning.
Take Home Message

For the purpose of local decision support:

• Climate model outputs can be misleading.

• New models and faster computers will not make the problem go away.

• We should focus on understanding and assessing uncertainty.

• This is best done using polling methods.
Thank you!